Abstract

On the existence theory for nonlocal nonlinear evolution equations with applications in peridynamics

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In this work, we prove existence of solutions to initial value problems for nonlocal, nonlinear evolution equations. The existence results are mainly applied to the peridynamic initial value problem, which is based on a nonlocal, nonlinear equation of motion of second order. We prove the following results.

(I) Under suitable LIPSCHITZ-type assumptions on the pairwise force function, there exists a unique classical solution to the peridynamic initial value problem. Both local-in-time and global-in-time results are proven.

(II) Modelling irreversible damage in peridynamics leads to a generalized initial value problem based on VOLTERRA operators. Under suitable LIPSCHITZ-type and boundedness assumptions on the pairwise force function, there exists a unique classical solution to the generalized nonlinear peridynamic initial value problem.

(III) By studying the limit of vanishing nonlocality of peridynamic energies, we identify the classical local energy of a material.

(IV) Under suitable growth and coercivity assumptions on a pairwise force function, that is strongly singular in its first argument, there exists a global weak solution to the nonlinear peridynamic initial value problem. The peridynamic operator neither is compact nor satisfies any (generalized) monotonicity condition. The proof of existence is based on a special GALERKIN scheme, the nonlocal character of the peridynamic operator, as well as the strucure of SOBOLEV–SLOBODECKIJ spaces.

(V) Modifying the arguments of the proof of result (IV), we present an alternative proof for the existence of a weak solution to the first order evolutionary fractional p-LAPLACIAN equation.

(VI) The doubly nonlinear second order evolution equation governed by two different fractional p-LAPLACIANs admits a weak solution.

(VII) Under suitable growth and coercivity assumptions on a pairwise force function, that is at most weakly singular in its first argument, there exists a YOUNG-measure-valued solution to the nonlinear peridynamic initial value problem.